



CEME Fellows Research Briefing Series — Volume 1, Issue 1

Assessing the Risk of Public Debt Schemes: An Asset-Liability Compound Option Approach ¹

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I. INTRODUCTION

The past years have witnessed profound changes in the way countries, especially in emerging markets, manage their public debt. Foremost among the changes has been increased reliance on arms-length financing through the issuance of government bonds, denominated either in foreign or domestic currency, in international and domestic markets. This development has been facilitated by the rapid growth of assets under management of institutional investors, especially long-term investors such as pension funds and insurance companies. In emerging market countries, such trend has fostered the development of domestic government and private securities markets.

In response to the changes highlighted above, the IMF and the World Bank developed and disseminated a number of principles aimed at ensuring sound public debt management.² Among them, three principles are directly related to ensuring risks in public debt schemes are properly managed. First, there should be adequate transparency and accountability enforced through disclosure of relevant risk measures (Principle 2). Second, debt structures should be easy to monitor and cost-efficient while minimizing liquidity and repayment risk (Principle 4). Third, there should be an appropriate risk management framework in place that accounts for the contingent-liability of the government (Principle 5).

Meeting the high standards set by the three principles listed above requires assessing the risks of public debt schemes from an asset-liability management perspective. This perspective can accommodate the risk from the currency composition and maturity mismatches of the asset and liability sides of the balance sheet, and more importantly, contingent liabilities such as those arising from public pension fund schemes guaranteed by the debt authority.

This paper describes a novel approach to asset-liability management of public debt schemes, the asset-liability compound option approach. This approach integrates standard asset-liability analysis with the option approach to model contingent liabilities. Furthermore, the paper also introduces a new measure for debt repayment capacity that is consistent with the definitions of the government balance sheet and net worth as presented in the 2001 IMF Government Finance Statistics. The rest of this paper is structured as follows. Section II presents the conceptual foundations of the model. Section III introduces the definition of debt repayment capacity. Section IV analyzes the risk profile of New Zealand's public debt. Section V concludes.

II. CONCEPTUAL FOUNDATIONS OF THE MODEL

The basic observation is that the value of the public sector net worth, in a multi-period setting, is equivalent to the value of a compound option, or more explicitly, the value of an option on an option on the total assets held by the government. In a multi-period setup, the debt issuer has the option to default at every debt repayment period. The decision to default depends on whether the call option on the asset value of the firm held by the debt issuer, or the net worth of the debt issuer, is “out-of-the-money.” In the context of public debt management, the net worth of the debt issuer is equivalent to the expected value of the current and future resources available to the debt agency or government to service the debt in a timely manner.

The observation above can be captured using the compound option pricing framework first introduced by Geske (1977). The compound option pricing framework improves on one-period option based models built on the work of Black and Scholes (1973) and Merton (1974). The calibration of these models requires knowing the equity price and volatility of the debt issuer, a set of information that is not available for a sovereign issuer or a public debt agency. More importantly, practical public debt management requires examining the cash flow profile of the debt profile. For instance, there are risks associated to the rollover of short-term debt that cannot be captured adequately in a one-period model. These problems could render one-period models impractical for real-world applications and/or policy advice.

The multi-period compound option framework, which relaxes the one period-restriction and accounts for the debt maturity and the timing of the cash flows associated to interest rate and principal payments, overcomes the problems listed above.³ Including the temporal dimension helps to identify, monitor and control risks associated with different primary surplus scenarios and their suitability vis-à-vis asset and liability matching needs (Togo, 2007). Furthermore, the intertemporal nature of the approach presented here enriches the standard macroeconomic approach to debt sustainability by adding information from market prices. In the next section, we explain in detail how to derive the net worth of the government, a required input into the ALM compound option framework.

III. THE GOVERNMENT BALANCE SHEET AND NET WORTH

According to the 2001 GFS Manual, the preferred measure for fiscal sustainability analysis is public sector net worth, which is defined as the difference between assets and liabilities at market prices and includes both financial and nonfinancial assets and liabilities. Several factors can lead to changes in net worth: (1) transactions associated with budget operations such as tax collection, social contribution, grants, and other revenues as well as the payment of salaries, goods, subsidies, grants, social benefits, interest, depreciation, and other expenses; (2) holding gains, and (3) changes in the volume of assets and liabilities unrelated to transactions and due to account reclassifications and exceptional or normal events.

Following Easterly and Yuravlivker (2000), Table 1 below shows the different items included in the public sector balance sheet. Such classification is consistent with the 2001 GFS Manual. Contingent liabilities, which are typically off-balance sheet items in corporate balance sheets, are also included. Among them, the most relevant categories are those associated with deposit insurance, pension fund schemes, and government debt guarantees to the private sector.⁴ The public sector inter-temporal budget constraint then requires that, in present value terms, the government’s net worth always exceed the operating balance, or government expenditures minus revenue.

Table 1. Public Sector Balance Sheet

Assets	Liabilities
Government-owned public goods (infrastructure, schools, health clinics, etc. that generate an adequate ERR and an indirect ERR through tax collection)	Public external debt
Government-owned capital that is financially profitable (anything for which government can charge user fees to generate adequate ERR)	Public domestic debt
Value of government-owned natural resource stocks (oil, minerals, etc.)	Domestic contingent liabilities (e.g. bank deposit guarantees, net present value of pension scheme, guarantees of private debt)
Expected present value of loans to private sector	Government's net worth

Source: Easterly and Yuravlivker (2000).

1/ ERR stands for economic rate of return.

Table 1 also shows that, in the public sector balance sheet, taxpayers are the government's "shareholders" since they are residual claimants. To see the analogy, note that taxes are equivalent to "retained earnings" and increase taxpayers' participation in the equity of the public sector.⁵ The distribution of good and services, as well as grants subsidies to taxpayers represent a dividend payment that reduces net worth. In the hypothetical liquidation of the government, the residual after paying creditors back would be distributed to taxpayers in the form of the provision of good and services.⁶

The realization that taxpayers hold a "shareholder" position in the public sector balance sheet implies that total debt, including domestic and external debt, needs to be considered when analyzing debt sustainability and in asset-liability management. In policy discussions, it is sometimes assumed that the residual claimants are domestic bond holders. Such assumption is wrong and could lead to an underestimation of the health of the government's financial conditions. In the next section we present an illustrative example.

IV. THE ALM COMPOUND OPTION APPROACH IN PRACTICE: RISK ANALYSIS OF NEW ZEALAND'S GOVERNMENT DEBT

The basic Geske (1977) compound option framework only includes two periods but it can be converted into a multiperiod model using Least Squares Monte Carlo Simulation. The intuition, however, can be explained in a simple two-period model as follows.⁷

Suppose that in the terminal period, $T = 2$, the government needs to pay a total amount M_2 that comprises both coupon and principal payments. The government chooses not to default if the value of the assets it holds, V_2 , exceeds what it owes. The value of the option not to default at $T = 2$ when valued in the first period, $T = 1$, depends on the value of the government assets in the first period, $C(V_1)$. The value of this option is the one that is compared with what the government owes in the first period, M_1 . If the option is worth more than what is owed,

the government services its debt in the first period and continues to the next period.

In our multi-period extension, we use the Least Squares Monte Carlo (LSMC) simulation method (Longstaff and Schwarz, 2001) to solve for the term structure of default probabilities and the one-year forward default probabilities of an ALM Compound Option model calibrated to New Zealand data as of end-June 2006. We choose the LSMC method because it is not affected by the upward or high bias associated to the use of plain Monte Carlo simulation for pricing American options.⁸

The New Zealand Crown uses accrual accounting since 1999 and complies with the 2001 IMF GFS manual recommendations since 2004. Assets are recorded at fair value and borrowings are recorded at book value. By end-2006, New Zealand's sovereign debt was rated investment grade, AA+ by Standard and Poor's, and AAA by Moody's. Information publicly available through web sites allows constructing annual data series for balance sheets, operating balances, domestic and external debt, pension benefits, and accident compensation claims.

Figure 1 illustrates the projections for debt service (assuming no currency swap effects), pension benefits and accident compensation claims for the 12 fiscal years following end-December 2006. Domestic debt comprises Kiwi and government bonds, Treasury bills, and loans. External debt includes debt issued in US dollars, Euros, Japanese yen, British pound, Norwegian Krona and Australian dollars. The government manages exchange rate risk actively, and frequently uses currency swap arrangements. As a result, the relative share of foreign currency-denominated flows increased in 2006 and allowed the government to benefit from the appreciation of the currency. As of June 2006, the fair value of net worth was NZD 669.5 million.

Because pension benefits and accident compensation claims are partially funded with public resources they do affect the probabilities of default of the sovereign. In the case of New Zealand, these contingent liabilities correspond to the Government Superannuation Fund (GSF) and the Accident Compensation Corporation (ACC). The projections in Figure 1 were based on publicly available information and incorporate assumptions for years with incomplete data. The current projections, therefore, are useful only for illustrative purposes and to show how contractual obligations affect the probabilities of default.

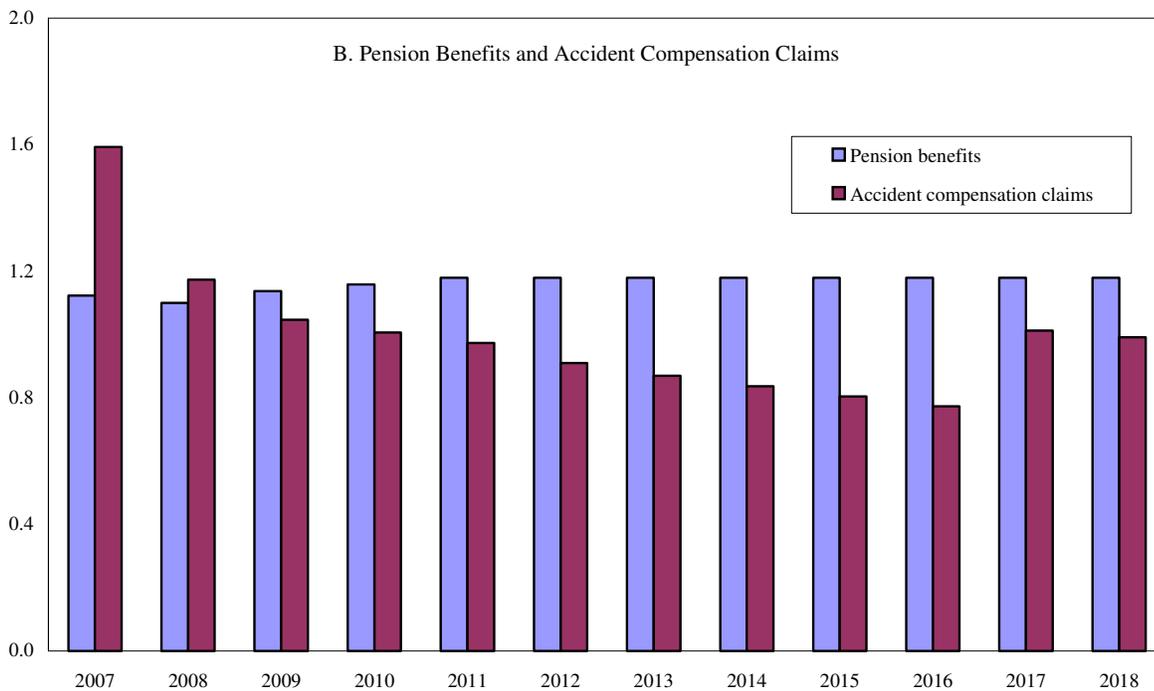
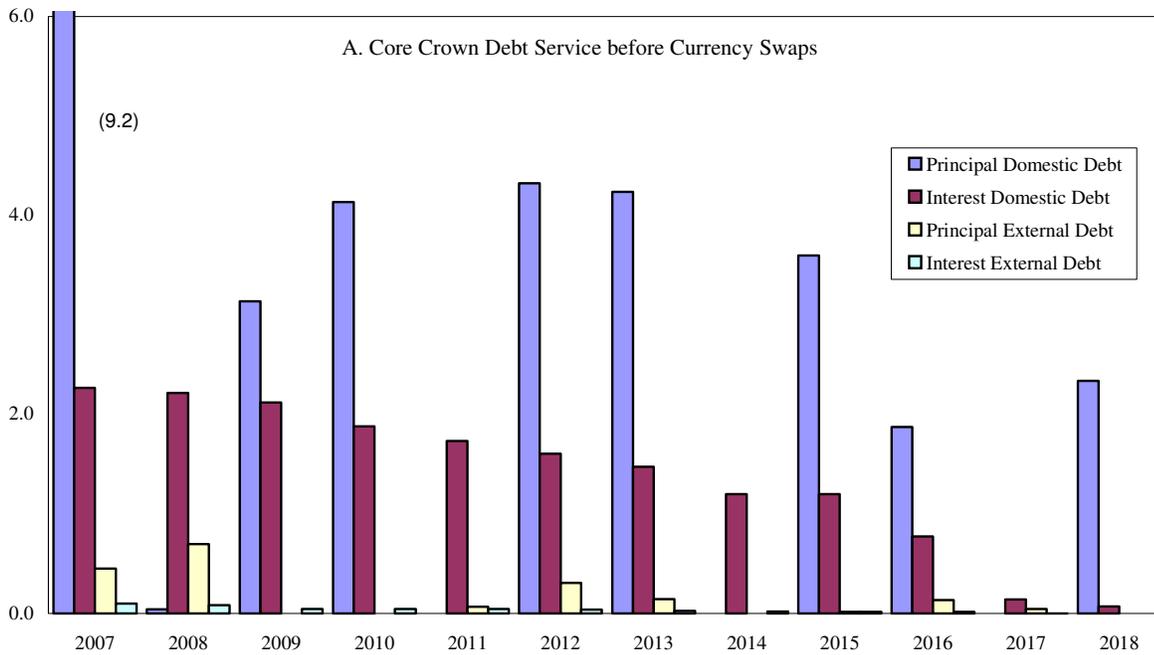
Solving the ALM compound option model requires estimates of the value of the assets in the Core Crown balance sheet and its volatility. These estimates were calibrated using the fair value of assets, as reported by the Core Crown, and its observed volatility estimated using maximum likelihood. The calibrated parameters were NZD 102.3 billion for the asset value, and 19.7 percent for the asset volatility.

In addition to asset volatility, the other risk factor in the balance sheet is associated with exchange rate movements. Although assets are denominated in multiple currencies, the analysis is simplified by considering only changes in the US dollar/NZD exchange rate on foreign-currency-denominated debt. By end-June 2006, the exchange rate was USD-NZD 1.6494. Maximum likelihood estimates of the exchange rate volatility and its correlation with assets were estimated as 14.8 percent and -0.393 respectively for the period 2000-2006.

The results obtained after performing 100,000 simulations are presented in Table 2. The table presents results corresponding to two different definitions of contractual obligations and the effects of one or two risk factors. The first definition only includes debt, the second definition adds pension benefits and accident compensation claims. The one-factor results include only asset volatility as a risk factor. The two-factor results include asset and exchange rate volatility as risk factors. The corresponding term structure of default probabilities and the one-year forward default probabilities are plotted in Figure 2.

Figure 1. New Zealand: Core Crown Debt Service, GFS Pension Benefits, and ACC Claims, End-June 2006

(In billions of New Zealand dollars)



Source: New Zealand Treasury, Government Superannuation Fund, Accident Compensation Corporation, and authors' calculations

Table 2. New Zealand: Core Crown Balance Sheets, June 2006

(In billions of New Zealand dollars)

	Book Value		Fair Value			
	End-June 2006	End-June 2006	One Risk Factor 1/ Debt		Two Risk Factors 2/ Debt	
	(1)	(2)	(3)	Debt, GFS, and ACC	(5)	Debt, GFS, and ACC
				(4)	(6)	
Assets	102.3	102.3	102.3	102.3	102.3	102.3
Liabilities	62.2	63.3	64.6	60.0	65.1	60.9
Borrowings	34.5	35.6	36.8		37.3	
ACC&GFS	27.7	27.7	27.7		27.7	
Net Worth	40.1	39.0	37.7	42.3	37.2	41.4

Source: New Zealand Treasury.

1/ Core Crown total assets as the only risk factor.

2/ Core Crown total assets and exchange rate as risk factors.

Either with one or two risk factors, the model provides a good approximation to the fair value of the liabilities and equity if debt is the only item included in the contractual obligations (columns 3 and 5 in Table 2). The slope of the term structure of probabilities of default is positive, which is typical for investment-grade issuers. Furthermore, it corresponds closely to the term structure of AA+ corporate default rates reported by Standard and Poor’s for the period 1981 – 2006 suggesting that the calibration yields a good fit.

The analysis of the two risk factors model including items other than debt under contractual obligations offers some interesting insights. The negative correlation between the exchange rate and the Core Crown asset value implies that adding the exchange rate as a second risk factor reduces volatility, which in turn, reduces the value of the option not to default. This reduction is equivalent to a reduction of taxpayers’ equity. Finally, the inclusion of pension benefits and accident compensations into contractual obligations increases the default risk of the public debt scheme. However, from the point of view of the taxpayer, the option of not defaulting on pension benefits and compensation claims has a positive value that is reflected in a higher net worth (columns 4 and 6 in Table 2).

V. CONCLUSION

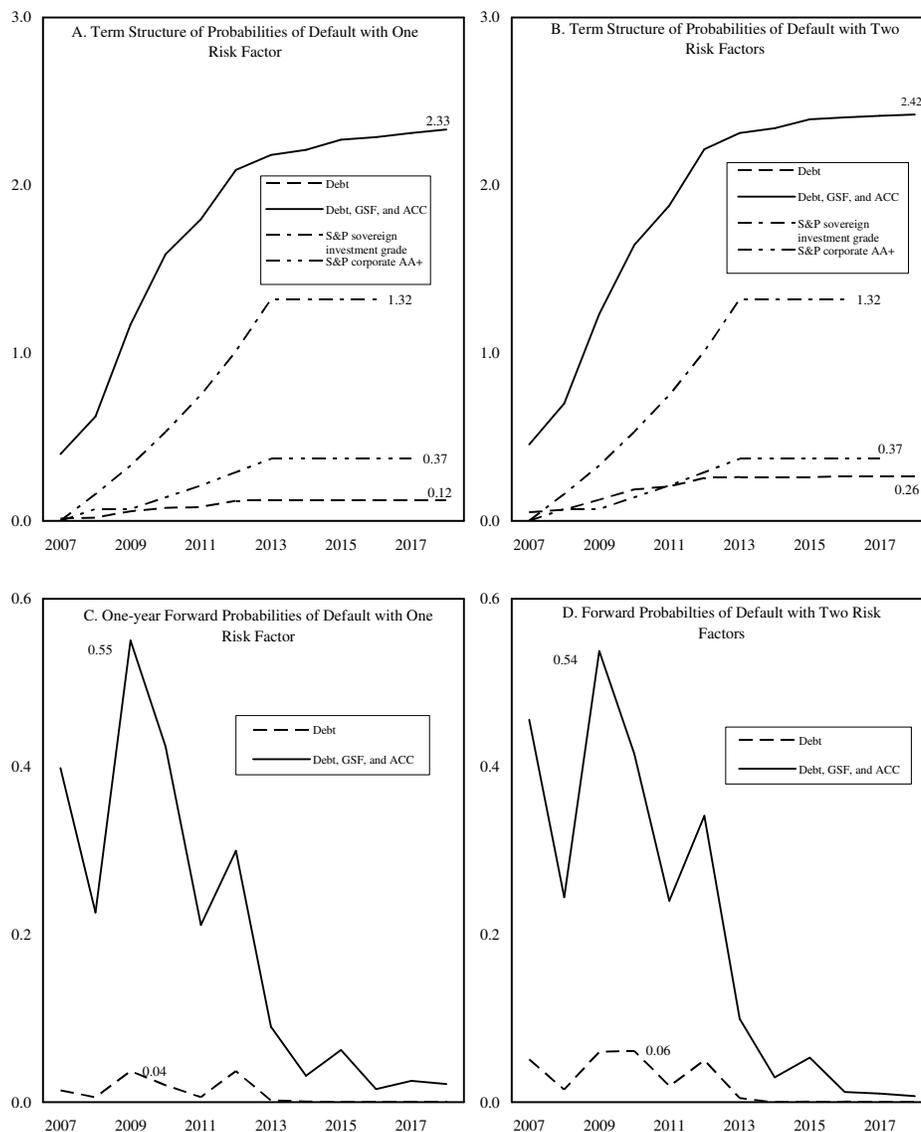
This paper presents a new approach, the ALM Compound Option Approach, to assess the risks associated with public debt scheme. As an illustrative example, the approach is used to examine the risk profile of New Zealand’s Core Crown office using one and two risk-factor models and different assumptions on what should be included as a contractual obligation.

The approach builds on the structural approach to credit risk analysis, where equity is viewed as a call option on the assets of the firm, and the macroeconomic approach to debt sustainability. In order to adapt the structural

approach to a multiperiod analysis, the ALM Compound option approach builds on the compound option pricing literature and more importantly, associates the identity of the “equity” holder with taxpayers in a manner consistent with standard government fiscal statistics, which facilitates the empirical implementation.

The example presented in this paper is based on the simplest implementation that assumes that asset prices are continuous, and interest rates, and volatility are constant. These assumptions can be relaxed however. For instance, asset prices can be modeled using jump-diffusions (Gukhal, 2004), interest rates can be stochastic (Chen, 2003), and stochastic volatility can be built in the model (Buraschi and Dumas, 2001). In addition, lines of credit with embedded options can be incorporated by building on the analytical solutions of extendible options (Longstaff, 1990) as well as government guarantees (see Merton and Bodie, 1992, and Draghi, Giavazzi, and Merton, 2003).

Figure 2: New Zealand—Core Crown Probabilities of Default, End-June 2006 (in percentage)



Source: New Zealand Treasury, Government Superannuation Fund, Accident Compensation Corporation, Bloomberg, and authors' calculations

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ENDNOTES

¹ This paper was written while the first author was affiliated with the International Finance Corporation, The World Bank Group. The views in this paper do not represent those of the IFC, the World Bank Group, or the IMF.

² See International Monetary Fund and International Bank for Reconstruction and Development (2003).

³ While the intuition builds on the Geske (1977) model, his closed form solution to the two-period problem cannot accommodate realistic debt profiles. Numerical solutions are required, as explained in section IV below.

⁴ One can also include contingent assets, such as credit lines to the private sector, in the government balance sheet.

⁵ Indeed, the New Zealand public sector balance sheets include the item “taxpayer fund” in net worth to account for the operating balance in a fiscal year.

⁶ Debt restructurings are more complex than this simplified description since bondholders and taxpayers share the restructuring burden. Usually, debt restructuring is conditioned on the adjustment programs as taxes are raised and expenditures reduced.

⁷ Chan-Lau and Santos (2009) provide full details on the model and simulation.

⁸ See Broadie and Glasserman (2004) and Glasserman (2004).

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